Benha University

Faculty of Engineering- Shoubra

Eng. Mathematics & Physics Department

Preparatory Year



Final Term Exam

Date: 17 - 5 - 2014

Course: Mathematics 1 - B

Duration: **3** hours

• Answer All Ouestions

• The Exam consists of one page

No. of questions: 4

• Total Mark: 100

### **Question 1**

(a)(i)Complete: The eigenvalues of a symmetric matrix of real numbers are...

(ii) State the types of solutions of a linear system AX = B.

(b)If 
$$A = \begin{bmatrix} 0 & -2 & -1 \\ 1 & 2 & 0 \\ 3 & 0 & -3 \end{bmatrix}$$
,  $B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 2 & -1 \end{bmatrix}$ . Find, if possible,  $A + B$ ,  $A.A$ ,  $A.B$ ,  $|A|$ ,  $|B|$  6

(c) If  $A = \begin{bmatrix} 2 & -1 \\ 4 & -3 \end{bmatrix}$ . Find (i) The eigenvalues and eigenvectors of A

(ii) The eigenvalues of  $f(A) = A^5 - 2A$  (iii)  $f(A) = A^n$ 

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# **Question 2**

(a) Solve the linear system: y - z = -3, x + 2y + 2z = 3, x + 3y + z = 0, 2x + y - z = -1

(b) Find S,  $S_{10}$  from: (i)  $\sum_{r=1}^{n} (r+1)(r+2)$  (ii)  $\sum_{r=1}^{n} \frac{1}{r^2+3r+2}$  (iii)  $\sum_{r=1}^{n} \frac{2r+3}{[(r+1)(r+2)]^2}$ 

(c)Using the mathematical induction, prove that:

(i) 
$$\frac{1}{1x^3} + \frac{1}{3x^5} + \frac{1}{5x^7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$
 (ii) If  $y = \frac{1}{1+2x}$ , then  $y^{(n)} = \frac{(-2)^n \cdot n!}{(1+2x)^{n+1}}$ 

## **Question 3**

(a) Write down the equation for a rotation of axis through an angle  $\pi/4$ . Hence prove that the curve  $2xy = a^2$  cane be transformed to  $x^2 - y^2 = a^2$ .

(b) Find the equation of the parabola with focus at (3, -4) and the directrix is 6x - 7y + 5 = 0.

(c) Find the equation of the ellipse whose foci  $(\pm 4, 0)$  and its eccentricity is 1/3.

(d) Find the equation of the circle with center at (6,6) and touch the circle  $x^2 + y^2 = 32$ .

## **Question 4**

(a) Find the equation of the two tangents of the hyperbola  $9x^2 - 4y^2 = 36$  drawn from the point (0,9). Find the angle between them.

(b)Prove that the circle  $x^2 + y^2 - 2ax - 2ay + a^2 = 0$  touch the axis.

Hence find the equation of the circle which touches the axis at a distance 4 from the origin.

(c) What conic the equation  $4x^2 - 9y^2 - 16x + 54y - 101 = 0$  represent? Find its foci and equation of its directrix.

Mid-Term Exam Algebra (12–

Algebra (12–4–2014)

Time: 1 hour T

Total mark: 20

Algebra

Group Section

No.

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[1]Complete the following statements:

- (a) A square matrix A is called symmetric if.....
- (b) A square matrix A is called singular if.....
- (c) A square matrix A has inverse if....

[2]If 
$$A = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$$
,  $B = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \end{bmatrix}$ .

- (a) Find if possible: A + B, A.B and B.A
- (b)Find the eigenvalues and eigenvectors of A
- (c) Find the eigenvalues of  $f(A) = A^3 2A$
- (d) Find the eigenvalues of  $f(A) = 2^A$
- (e)Find A<sup>n</sup>

#### Benha University

Faculty of Engineering- Shoubra

Eng. Mathematics & Physics Department

Preparatory Year



Final Term Exam

Date: 14 - 6 - 2014Course: Mathematics 1 - B

Duration: **3** hours

• Answer All Questions

• The Exam consists of one page

(تخلفات)

No. of questions: 4 Total Mark: 100

### **Question 1**

(a)If 
$$A = \begin{bmatrix} 0 & -2 & -1 \\ 1 & 2 & 0 \\ 3 & 0 & -3 \end{bmatrix}$$
. Show that  $A + A$ ,  $A$ . are symmetric matrices and find  $|A|$ 

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(b)If  $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ .

(i) Find the eigenvalues and eigenvectors of A (ii) Write and satisfy Hamilton equation

(iii) Find the eigenvalues of  $B = A^2 - 2A$ 

(c) Solve the linear system: x + 2y - 2z = 2, 2x + y + 3z = 2, 3x + 3y + z = 4.

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### **Question 2**

(a) If 
$$z_1 = 2 - i$$
,  $z_2 = -1 + 2i$ . Find  $z_1 \cdot z_2$ ,  $z_1/z_2$  and  $(z_1 + z_2)^9$ 

(b) Find S,  $S_{12}$  from each series: (i)  $\sum_{r=1}^{n} r^2 (2r-1)$  (ii)  $\sum_{r=1}^{n} \frac{2}{4r^2-1}$ 

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(c)Using the mathematical induction, prove that:

(i)  $\frac{1}{2r5} + \frac{1}{5r8} + \frac{1}{8r11} + \cdots + n - \text{term} = \frac{n}{6n+4}$ (ii)  $n^3 + 2n$  is divisible by 3

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(d) Using the binomial theorem, expand:  $\frac{1}{3x-2}$ 

## **Question 3**

(a) Find the equations of straight lines bisecting the angle between the pair of lines:  $4x^2 - 24xy + 11y^2 = 0$ .

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(b) Find the equation of the tangent to the circle:  $x^2 + y^2 - 6x + 4y - 12 = 0$  which parallels to the line: 4x + 3y + 5 = 0.

(c) Find the equation of the parabola where its focus at (0, 3) and D: x = 1.

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## **Question 4**

(a) Find the latus rectum, eccentricity and the foci of the ellipse:  $x^2 + 2y^2 + 4x + 4y - 2 = 0$ . 9

(b) Find the equation of hyperbola whose eccentricity 5/4 and focus at (a, 0) and directrix 4x - 3y - a = 0.

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(c) Find the equation of the plane passing through the line: 2x + 3y + 4z = 16, 4x + y + 6z = 14 and parallels to the plane: x - y + z + 6 = 0.

Good Luck.

Dr. Ibrahim Sakr

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